PRELIMINARY STUDY ON A SIMPLIFIED RESPONSE SPECTRUM METHOD FOR INCOHERENT GROUND MOTIONS OF BRIDGES

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Cover graphic: Depicts the pseudo-static displacement response in the simplified two degree-of-freedom model used to study the effects of incoherent ground motion. Unlike uniform base motion, the different displacement history response at each support during an earthquake will induce forces within a bridge.
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# TABLE OF CONTENTS

EXECUTIVE SUMMARY...........................................................................................................vii

CHAPTER 1 - INTRODUCTION ..............................................................................................1
1.1 Introduction ......................................................................................................................1
1.2 Sources of Incoherency ...................................................................................................2
1.2.1 Response History Analysis Methods.................................................................3
1.2.2 Response Spectrum Analysis Method...............................................................4
1.3 Review of Previous Work ..............................................................................................4
1.4 Object and Scope..........................................................................................................5

CHAPTER 2 - ANALYTICAL MODELS ..............................................................................7
2.1 Introduction ....................................................................................................................7
2.2 Two Degree-of-Freedom System ..................................................................................8
2.2.1 Equation of Motion Formulation..........................................................................9
2.2.2 Influence Functions ..............................................................................................11
2.2.3 Modal Response History Method.........................................................................12
2.2.4 Model Verification ..............................................................................................14
2.3 Ground Response Model ...........................................................................................15
2.3.1 Equivalent Linear Method..................................................................................16
2.3.2 Nonlinear Analysis Methods..............................................................................17
2.3.4 Model Verification ..............................................................................................20
2.4 Response Spectrum Method ......................................................................................21

CHAPTER 3 - PARAMETRIC STUDIES ......................................................................23
3.1 Introduction ..................................................................................................................23
3.2 Coherency Function Models ......................................................................................24
3.3 Frequency Parameters ...............................................................................................25
3.3.1 Coupling Ratio ...................................................................................................26
3.3.2 Ground Motions and Soil Parameters ...............................................................27
3.4 Results ........................................................................................................................28
3.5 Limitations of the Parametric Study .........................................................................29

CHAPTER 4 - SUMMARY AND CONCLUSIONS .......................................................31
4.1 Summary ......................................................................................................................31
4.2 Conclusions ................................................................................................................32
4.3 Future Research .........................................................................................................33

REFERENCES.....................................................................................................................35

APPENDIX A - MAGNIFICATION FACTORS
APPENDIX B - COMPUTER MODEL SOURCE CODE
LIST OF TABLES

Table 2.1 - Parameters used to verify the computer model .............................................. 36
Table 3.1 - Masses used for the 2 DOF model................................................................. 36
Table 3.2 - Shear beam stiffnesses used for the 2 DOF model........................................... 37
Table 3.3 - Soil deposit depth combinations ..................................................................... 37
LIST OF FIGURES

Figure 2.1 - 2 DOF model ............................................................................................................. 38
Figure 2.2 - Pseudo-static displacement response ......................................................................... 38
Figure 2.3 - Dynamic displacement response comparison at DOF 1 .................................................. 39
Figure 2.4 - Total displacement response comparison at DOF 1 ..................................................... 39
Figure 2.5 - Ground motion response model .................................................................................. 40
Figure 2.6 - Variation of damping ratios for sands .......................................................................... 41
Figure 2.7 - Variation of shear modulus for sands .......................................................................... 42
Figure 2.8 - Theoretical primary curve for a sand deposit ............................................................... 43
Figure 2.9 - Primary curve used in the computer model for a sand deposit ..................................... 43
Figure 2.10 - Ground motion response - Northridge earthquake (Scale factor = 1) ................. 44
Figure 2.11 - Ground motion response - Northridge earthquake (Scale factor = 4) ............... 44
Figure 3.1 - Predicted absolute ground acceleration : Taft earthquake ........................................ 45
Figure 3.2 - Predicted absolute ground acceleration : Taft earthquake ........................................ 45
Figure 3.3 - Predicted absolute ground acceleration : El Centro earthquake ................................. 46
Figure 3.4 - Predicted absolute ground acceleration : El Centro earthquake ................................. 46
Figure 3.5 - Predicted absolute ground acceleration : Northridge earthquake ............................. 47
Figure 3.6 - Predicted absolute ground acceleration : Northridge earthquake ............................. 47
Figure 3.7 - Predicted relative ground displacement : Taft earthquake ......................................... 48
Figure 3.8 - Predicted relative ground displacement : Taft earthquake ......................................... 48
Figure 3.9 - Predicted relative ground displacement : El Centro earthquake ............................... 49
Figure 3.10 - Predicted relative ground displacement : El Centro earthquake .............................. 49
Figure 3.11 - Predicted relative ground displacement : Northridge earthquake .......................... 50
Figure 3.12 - Predicted relative ground displacement: Northridge earthquake ............... 50

Figure 3.13 - Average magnification factors: Taft earthquake ........................................ 51

Figure 3.14 - Average magnification factors: El Centro earthquake ............................... 51

Figure 3.15 - Average magnification factors: Northridge earthquake ......................... 52

Figure 3.16 - Average magnification factors: All earthquakes ........................................ 52
EXECUTIVE SUMMARY

Many instances arise in bridge design where non-synchronous ground motions occur at different support locations. Although several sources of incoherent ground motion are recognized, differing soil conditions at intermediate supports along a bridge can occur frequently. Several methods are available to predict bridge response to incoherent ground motions, but are typically cost prohibitive to implement in standard bridge designs. Unlike a bridge that is excited with uniform ground motion, the behavior of bridges undergoing non-synchronous ground motions are not well understood. For bridges subjected to incoherent ground motions, both the ground acceleration and displacement contribute to the overall bridge response. To better understand bridge response due to the site-response effect, a study funded by the EERI/FEMA Professional Fellowship was undertaken.

To simplify the analysis and understand the pseudo-static displacement and acceleration behavior, a response history computer model for a two degree-of-freedom (DOF) system was developed. A shear beam connected both DOF to represent the superstructure of a bridge. Superposition of the pseudo-static and acceleration response was used to determine the total DOF response. A SDOF, nonlinear ground response model was used to predict the site-response effect. An iteration scheme using the Newton-Raphson method was implemented and verified with an equivalent linear stiffness method. Parametric studies were conducted considering different structure masses and shear beam stiffnesses. Three different earthquake ground motions were used and the soil depths were varied at the supports. A stiffness parameter, defined as the Coupling Ratio (CR), was defined to quantify the effect of the shear beam stiffness on the
overall structure behavior. CR varies from 0 (each DOF responds independently) to 1 (rigid body response). Magnification factors were calculated as the ratio of the relative displacement at each DOF due to the incoherent ground motion to the uniform base motion response. The magnification factors can be used to account for ground motion incoherency by modifying the response of a two DOF system subjected to uniform base motion.
CHAPTER 1

INTRODUCTION

1.1 Introduction

Many instances arise in bridge design that cause differing earthquake ground motions at supports of a structure. Accurate estimates of seismic induced forces are critical to provide reliable designs for superstructure and substructure elements. To simplify the seismic force calculations, design codes assume that all supports have the same intensity of ground shaking, although many bridges have conditions that modify the intensity and frequency content of seismic shaking (Ref. 1). Bridge responses to incoherent earthquake ground motions are due to a dynamic portion and differing support movements (denoted as “pseudo-static” displacements). Due to the nature of non-synchronous effects on structures, it is not apparent whether the dynamic or pseudo-static response will increase bridge forces as compared to uniform base motions. Some studies suggest incoherency cancels peak amplitudes and reduces force levels. Another study indicates that the pseudo-static response increases force levels in stiff structures.

One method to determine the effects of incoherent ground motion has been developed using response history analysis. Using this method, the analytical model is subjected to differing earthquake ground motions and the response histories of the bridge elements are calculated. A drawback of the response history method is determining representative earthquake ground motions for the site and different supports. Depending upon the site and bridge length, considerable costs can be involved in developing ground motions that encompass the design earthquake. Typical bridge designs for short and medium span lengths do not use response history analysis due to the increase in cost to
develop and implement site-specific ground motions. The most common method used to
determine dynamic response in bridges is the response spectrum method (RSM).
Widespread acceptability of the RSM is due to the relative simplicity in definition of the
design earthquake and the use of the linear elastic analysis to simulate actual behavior
during earthquakes. The RSM currently used does not account for non-synchronous
ground motions.

1.2 Sources of Incoherency

Four different sources of incoherency (Ref. 5) are typically recognized in seismic ground
motions: 1) the loss of coherency due to the scattering of seismic waves and the
superposition of waveforms from differing sources (denoted as the “incoherency effect”),
2) difference in arrival times of the waveform as they propagate through the underlying
strata (denoted as the “wave passage effect”), 3) decay of wave amplitude with distance
due to spreading and energy dissipation (denoted as the “attenuation effect”) and 4)
variation of waveforms emanating from a similar bedrock layer through differing soil
strata, which modify the amplitude and frequency of the waveforms at the base of the
structure (denoted as the “site-response effect”). The attenuation effect has been shown
to have limited influence on the coherency function and can be ignored (Ref. 6). In this
study, only the site-response effect will be considered as the source of non-synchronous
ground motion. It is believed that other effects can be critical only for relatively long
bridges.
1.2.1 Response History Analysis Methods

Response history analysis methods have been developed and are available to estimate the effects of incoherent ground motion (Ref. 3). The equations of motion can be written to include the contribution of individual supports to the degrees-of-freedom within the structure. A complete formulation of the equations of motion to include non-synchronous ground motions will be presented in Chapter 2. Fenves (Ref. 4) used the computer program SAP90 (Ref. 5) with response history capabilities to study the Dumbarten Bridge and compare analytical to measured response due to the Loma Prieta earthquake. Because SAP90 does not have the capability to include the effects of non-synchronous ground motions, a fictitious spring at each support was introduced to model the incoherency using time varying loads. Currently, state-of-the-art software packages, such as SAP2000 (Ref. 6), are able to account for differing support ground motions using the free-field ground displacement histories as input to the analytical model. Results of the study indicate that the Dumbarten Bridge was not sensitive to incoherent ground motion, but indicate that the pier displacements increased somewhat as compared to uniform base motion. This study was limited to incoherency due to the site-response effect. It was concluded that the non-synchronous ground motions did not substantially increase forces levels for that particular bridge. Due to similar ground displacement and acceleration histories along the length of the bridge at different supports, the overall bridge response should not differ greatly from the uniform base motion response.
1.2.2 Response Spectrum Analysis Method

The response spectrum method (RSM) is used in practice to conduct bridge dynamic analysis. An inherent assumption of the RSM is that all supports of the structure are excited with the same earthquake ground motion. Der Kiuregian (Ref. 5) has developed one method to estimate the response to incoherent motion called the Multiple Support Response Spectrum (MSRS) rule. This method was developed based on random vibration theory and solution of the governing equations of motion in the frequency domain instead of the time domain. All sources of differing ground motions are depicted using frequency-based coherency functions, and structure response quantities are determined using correlation coefficients to determine support contribution of response due to each degree of freedom. The MSRS rule has significant computation requirements to determine the cross correlation coefficients. Results from two separate parametric studies conducted on four span bridges indicate that 1) an envelope of the site response spectrums can be unconservative and should not be used to estimate the seismic response, and 2) a single-degree-of-freedom ground response model can reasonably predict site response.

1.3 Review of Previous Work

Past studies of incoherent ground motions have mainly been project specific using response history analysis methods. Some researchers have been developing simplified models to better understand the components of bridge behavior under non-synchronous ground motions. Mylonakis has studied a simplified model with sinusoidal support movements and presented observations (Ref. 2). Studies on multi-span bridges have been
limited to the longitudinal axis assuming that the deck is rigid. Very few in-depth studies have been conducted to determine the effect of incoherent ground motions on bridges and to develop methods or design guidelines that can be readily implemented in design. The remainder of this section will discuss response history analysis methods, and response spectrum methods.

1.4 Object and Scope

The main objective of this study was to provide the basis for a practical RSM that can account for non-synchronous ground motions to estimate bridge response due to the site-response effect. Another goal of this study was to provide insight into bridge response due to incoherent ground motions using a simplified computer model. To study the non-synchronous effects, a two-degree-of-freedom computer model using response history analysis was developed. A nonlinear, SDOF ground response model was developed to estimate the site-response effect. Using the results of parametric studies, factors were calculated that compare the incoherent to the uniform base motion response. The factors can be applied to a typical response history analysis results for uniform base motion to estimate the bridge response under incoherent ground motion. Parameters varied in the study were the mass and shear beam stiffness, while three different earthquakes and multiple soil depths were used.

Chapter 2 describes the two-degree-of-freedom computer model used in the study. Development of the non-linear and equivalent linear SDOF ground response model is also presented in Chapter 2. Chapter 3 describes a brief overview of a coherency function model to provide insight into the complexities of incoherent ground motions.
Chapter 3 also discusses the parametric study variables used for the bridge model and site conditions, and presents the results and observations from the study. A summary of the study, important conclusions and recommendations for future work are presented in Chapter 4. An appendix has been included with graphs that present the magnification factors developed in the parametric study. Another appendix is included with portions of the computer code developed for the two-degree-of-freedom model and the non-linear SDOF ground response model.
2.1 Introduction

To estimate the response of bridges during earthquakes, several different methods are available. As a state-of-the-practice, a linear elastic analysis is conducted using the Response Spectrum Method (RSM). An inherent assumption in the RSM is that all the supports of the structure are subjected to the same ground motion. For more complicated or important bridges, response history analysis may be used to determine the seismic response. Because force levels in bridges may be affected by non-synchronous ground motion, an accurate response analysis should be conducted to properly size bridge members and foundations.

Structural analysis software has not been readily available to estimate incoherent ground motion response. One researcher used fictitious springs at the supports to simulate the pseudo-static support displacements (Ref. 4). Recently, finite element analysis packages have been developed to calculate the dynamic response due to incoherent ground motions using the response history method (Ref. 7). A response history analysis is typically not feasible to produce cost-effective bridge designs for ordinary span length bridges. A method that has been proposed to conduct dynamic analysis using incoherent ground motions is the Multiple Support Response Spectrum (MSRS) rule (Ref. 7). Due to the complexity in determination of different factors and input ground motion response, this method has not received widespread use in bridge design. As a simplified method, several researchers have suggested that the site response coefficients may be averaged to account for incoherency (Ref. 8).
To study the effects of incoherency, a simplified two degree-of-freedom (DOF) system consisting of two masses and springs connected by a shear beam was developed. To gain a better basic understanding of bridge response, a detailed finite element model of different bridges was not considered. In-depth parametric studies that vary the bridge mass and stiffness and the soil depths were conducted and will be presented in Chapter 3. In this chapter, the two DOF system connected with a shear beam will be discussed. Development of the equations of motion will be presented using the superposition method and solution using the displacement histories. Influence functions will be derived for a generalized structure and applied to the simplified model. Development of the modal response history method using modified mode participation factors will be presented. A single DOF soil column response model was developed using a non-linear and equivalent linear method and will be presented. An overview of the response spectrum method will be discussed.

2.2 Two Degree-of-Freedom System

To understand and study the effects of incoherent ground motion, a two DOF model connected with a shear beam was developed and shown in Figure 2.1. A 2 DOF model was selected for study to reduce the complexities of the analysis and allow the pseudo-static and dynamic response components to be studied. Each DOF in the model consists of a mass connected by a support spring and a shear spring. The support springs represent the substructure of the bridge including the foundation springs. Superstructure stiffness is represented with the shear beam spring. As stated previously, this study will be limited to only the site-response effect.
2.2.1 Equation of Motion Formulation

To determine the response of a structure due to incoherent earthquake ground motions, the total response can be determined from a superposition of the dynamic response and the pseudo-static ground displacement response. An alternative formulation of the governing equations of motion can be solved using the support ground displacement history. Using the superposition method, the governing equations for the dynamic response of the structure due to ground acceleration at support \( n \) can be described as

\[
\begin{bmatrix}
M
\end{bmatrix}\ddot{x}_{\text{dyn}} + \begin{bmatrix}
C
\end{bmatrix}\dot{x}_{\text{dyn}} + \begin{bmatrix}
K
\end{bmatrix}x_{\text{dyn}} = -\begin{bmatrix}
M
\end{bmatrix}\begin{bmatrix}
r
\end{bmatrix}\ddot{x}_{gn}
\]

(2.1)

in which \( M \) is the mass matrix, \( C \) is the damping matrix, \( K \) is the stiffness matrix, \( \ddot{x}_{\text{dyn}}, \dot{x}_{\text{dyn}}, x_{\text{dyn}} \) are the structure acceleration, velocity and displacement, respectively, relative to the undeflected position, \( r \) is the influence factor matrix and \( \ddot{x}_{gn} \) is the ground acceleration at support \( n \). Equation 2.1 can be solved using a direct integration approach such as Newmark’s Beta Method or superposition of a modal analysis. For this study, the equations of motion was solved using modal analysis. The pseudo–static displacement response at DOF \( i \) due to support \( n \) can be determined by partitioning of the stiffness matrix into nodes at DOF and support nodes as

\[
\begin{bmatrix}
K_{ff} & K_{fs} \\
K_{sf} & K_{ss}
\end{bmatrix}
\begin{bmatrix}
U_f \\
U_s
\end{bmatrix}
= \begin{bmatrix}
P_f \\
P_s
\end{bmatrix}
\]

(2.2)

in which the subscripts \( f \) denote a DOF and \( s \) denotes a support node. Assuming that external loads are not applied to the structure, the displacement at DOF \( i \) can be determined due to a unit displacement at support \( n \) described as
\[ K_{ff} U_{ff} + K_{fs} U_{ss} = 0 \]  \hspace{1cm} (2.3)

Equation 2.3 was developed using the first matrix equation in equation 2.2. Solving equation 2.2.3 for the displacement at the free node

\[ U_{st} = U_{ff} = -K^{-1}_{ff} K_{fs} U_{ss} \]  \hspace{1cm} (2.4)

in which \( U_{st} \) are the pseudo-static displacements. Equation 2.4 describes the displacement at the structural DOF due to support displacements (denoted as “pseudo-static displacement”) and shown in Figure 2.2. For the case in which \( U_{ss} \) is equal to a unit displacement, \( U_{ff} \) is defined as the influence functions and will be developed for the 2 DOF system in the following section. The total response at the structural DOF can be determined using superposition

\[ U_T = U_{dyn} + U_{st} \]  \hspace{1cm} (2.5)

in which \( U_T \) are the total displacements. An alternate formulation of the governing equations of motion can be developed using

\[ [M] \ddot{U}_T + [K] U_{dyn} = 0 \]  \hspace{1cm} (2.6)

in which damping effects are considered negligible. Substituting equation 2.5 into equation 2.6

\[ [M] \ddot{U}_T + [K] U_T = [K] U_{ss} \]  \hspace{1cm} (2.7)

Solution of equation 2.7 will provide the total displacement at the structural degrees-of-freedom due to dynamic and pseudo-static response. In equation 2.7, \( U_{ss} \) must be defined as the total bedrock and free-field ground displacement history. Otherwise, the dynamic contribution will not include the response due to the bedrock ground motion.
2.2.2 Influence Functions

The influence functions are defined as the displacement at DOF $i$ due to a unit displacement at support $n$ and can be determined using equation 2.4. For the two DOF system, the stiffness matrix can be developed

$$K = \begin{bmatrix}
1 & K_1 & -K_1 \\
2 & -K_1 & K_1 + K_{SB} & -K_{SB} \\
3 & -K_{SB} & K_2 + K_{SB} & -K_2 \\
4 & -K_2 & -K_2 & K_2
\end{bmatrix}$$  \hspace{1cm} (2.8)

in which $K_1$ and $K_2$ are the support spring stiffnesses and $K_{SB}$ is the shear beam stiffness.

Rearranging the matrix to form the partitions defined in equation 2.2

$$K = \begin{bmatrix}
2 & K_1 + K_{SB} & -K_{SB} & -K_1 \\
3 & -K_{SB} & K_2 + K_{SB} & -K_2 \\
1 & -K_1 & K_1 & -K_2 \\
4 & -K_2 & K_2 & K_2
\end{bmatrix}$$  \hspace{1cm} (2.9)

Note that nodes 2 and 3 are the DOF and nodes 1 and 4 are the restraints. Using the partitioned sub-matrices from equation 2.9 and substituting into equation 2.2 the influence function matrix is defined as

$$r = \begin{bmatrix}
2 & K_1(K_2 + K_{SB}) & K_2K_{SB} \\
3 & K_1K_{SB} & K_2(K_1 + K_{SB})
\end{bmatrix} \begin{bmatrix}
1 \\
\frac{1}{K_1K_2 + K_2K_{SB} + K_1K_{SB}}
\end{bmatrix}$$  \hspace{1cm} (2.10)

A property of the influence matrix is

$$\sum_{n=1}^{n_{tot}} r_{in} = 1.0$$  \hspace{1cm} (2.11)

in which the subscript $i$ denotes the DOF, $n$ denotes the support and $n_{tot}$ are the total number of supports. It should also be noted that the ground acceleration distributes from
an individual support to the structural DOF using the influence function matrix due to the fact that the acceleration is a function of the displacement.

### 2.2.3 Modal Response History Method

Unlike the direct integration response history analysis method in which the governing equations are solved to determine the total response of the DOF, in modal analysis the response of individual modes are superimposed to determine the total response. By assuming that damping effects are negligible, the ground acceleration is zero, and defining \( \ddot{x} = -\omega^2 x \), equation 2.1 can be reduced to

\[
\left[ K - \omega_n^2 M \right] \phi_n = 0 \tag{2.12a}
\]

\[
\det(K - \omega_n^2 M) = 0 \tag{2.12b}
\]

in which \( \omega \) is the natural circular frequency for mode \( n \), and \( \phi \) are the modal displacements or mode shape at DOF \( i \) for mode \( n \). Equation 2.12b will yield a non-trivial solution to equation 2.12a in which modal frequencies and modal displacements can be calculated. To determine the response of individual modes, the displacement at DOF \( i \), \( x_i \), can be described as

\[
x_i = \sum_{n=1}^{N} \phi_m^T \phi_n \zeta_n \tag{2.13}
\]

in which \( \zeta \) is the displacement response of mode \( n \). To determine the dynamic response for mode \( n \), equation 2.13 is substituted into equation 2.1 and multiplied by \( \phi_m^T \)

\[
\phi_m^T m \left( \sum_{n=1}^{N} \phi_n \zeta_n \right) + \phi_m^T c \left( \sum_{n=1}^{N} \phi_n \dot{\zeta}_n \right) + \phi_m^T k \left( \sum_{n=1}^{N} \phi_n z_n \right) = -\phi_m^T \Gamma \zeta_g \tag{2.14}
\]
in which equation 2.14 and equation 2.1 are termed the coupled equations of motion, which refers to the fact that the solution must be obtained by direct integration of all equations of motion. To uncouple the equations of motion such that the dynamic response of the structure can be determined using the superposition of results from individual modes, orthogonality relationships can be developed

\[ \sum \phi_m^T m \phi_n = 0 \]  
(2.15)

\[ \sum \phi_m^T c \phi_n = 0 \]  
(2.16)

\[ \sum \phi_m^T k \phi_n = 0 \]  
(2.17)

Equations 2.15 through 2.17 are valid when \( m \neq n \). Applying the orthogonality relationships in equations 2.15 through 2.17 to equation 2.14 and dividing the resulting equation by the modal mass, \( \phi_m^T m \phi_n \),

\[ \ddot{z} + 2\xi_i \omega_i \dot{z} + \omega_i^2 x = -\gamma_p m \ddot{x}_g \]  
(2.18)

in which \( \xi \) is the damping coefficient for mode \( i \), and \( \gamma_p \) are the modified mode participation factors defined as

\[ \gamma_p = \left( \frac{\phi_m^T m r}{\phi_m^T m \phi} \right) \]  
(2.19)

For uniform base motion, \( r \) is defined as a matrix vector of 1’s and corresponds to the uniform distribution of acceleration to all DOF. For the case in which non-synchronous ground motions are considered, \( r \) is defined using equation 2.10. Total response for the structure can be determined using equation 2.5 when a response history analysis is
conducted. Equation 2.18 can be recognized as the response of a single degree-of-freedom (SDOF) oscillator. When non-synchronous ground motion response is considered, equation 2.18 will further reduce to the SDOF response due to individual support ground motions. The total number of SDOF oscillators to be considered are dependant on the number of modes to be included in the analysis and the number of supports with similar input ground motions.

2.2.4 Model Verification

A Windows™ based computer program, IGMR (Incoherent Ground Motion Response), was developed to calculate the total response due to incoherent ground motions using the response history analysis method. A superposition of the dynamic and pseudo-static displacement response using modal analysis was implemented. To verify the accuracy of the computer implementation, results from IGMR were compared with SAP2000 (Ref. 7). To conduct an incoherent ground motion response analysis using SAP2000, the following steps are done:

1) Apply unit displacements at each support with similar time history functions. SAP2000 will numerically evaluate the influence function matrix, and

2) Assign displacement time histories to the supports. SAP2000 uses the displacement response history method in equation 2.7 to determine the total DOF response.

Parameters used in the structural model are listed in Table 2.1. Bedrock ground motion used was the 1952 Taft earthquake. Figure 2.3 depicts the displacement response history at DOF 1 for the uniform ground motion case using a 50-foot deep soil deposit and indicates excellent agreement. Free-field ground motion displacement histories were
applied to the different supports in SAP2000 using the above procedure. Figure 2.4 depicts the displacement response history using non-synchronous ground motion due to a soil column of 50-feet at support 1 and 150-feet at support 2. Displacement response histories calculated show good agreement. Minor differences in peak displacement can be noted and are attributed to inaccuracies that arise in the numerical integration.

2.3 Ground Response Model

Four different sources of incoherency were briefly presented in Chapter 1, although for this study, only the site-response effect will be considered. To determine the free-field ground response, the following assumptions were made:

1) Bedrock ground motion at each support are the same,
2) Underlying soil strata can be defined as a shear beam, and
3) Soil deposit response is due to vertically propagating shear waves.

To estimate the free-field ground motion response, a SDOF model was developed and implemented into IGMR. One study recommends that a MDOF model should be used and divided into numerous layers depending upon the fundamental period of the deposit (Ref. 9). These findings were developed based upon comparison of results from a lumped mass model to a distributed parameter model. Past studies (Ref. 7) have shown that the SDOF soil model reasonably approximates a more detailed analysis. Figure 2.5 depicts the ground motion response model. The equation of motion of the soil mass can be described as

\[
\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x = -\ddot{x}_s
\]  

(2.20)
in which \( \ddot{x}, \dot{x}, x \) are the relative ground acceleration, velocity and displacement, respectively, as compared to the bedrock, \( \zeta \) is the damping ratio, \( \omega \) is the natural circular frequency of the deposit, and \( \ddot{x}_s \) is the bedrock acceleration. Using a lumped mass assumption for the SDOF model, \( \omega \) can be defined as

\[
\omega = \sqrt{2} \frac{V_s}{H}
\] (2.21)

in which \( V_s \) is the shear wave velocity of the deposit and \( H \) is the depth of the soil deposit. \( V_s \) can be defined from \( G = \rho V_s^2 \), in which \( G \) is the shear modulus and \( \rho \) is the soil density. Theoretical studies (Ref. 9) have shown that the frequency of the soil deposit with distributed parameters is defined as

\[
\omega = \left( \frac{\pi}{2} \right) \frac{V_s}{H}
\] (2.22)

Within a soil deposit, the damping ratio and shear modulus are nonlinear functions that vary with shear strain. The experimental data used in this study are shown in Figures 2.6 and 2.7. The damping ratio represents the hysteretic damping due the shear strain in the deposit. To estimate the free-field response of the soil deposit, equation 2.20 can be solved using an equivalent linear or nonlinear analysis method.

### 2.3.1 Equivalent Linear Method

Due to complexities in nonlinear analysis, Seed and Idriss (Ref. 9) proposed an equivalent linear method to predict soil deposit response. Both the soil deposit stiffness and damping in this model can be represented using an equivalent spring and damper
based on the maximum shear strain within the soil deposit. The equivalent linear method can be conducted using the following steps:

1) Assume a shear strain, $\gamma$, in the soil deposit,

2) Determine $G_i$ and $\varepsilon_i$ based on $\gamma$,

3) Calculate $\omega_i$ using equation 2.22,

4) Calculate the response history using equation 2.20 and determine $\gamma_{\max}$,

5) Determine the average cyclic shear stress, $\gamma_{\text{avg}} = \gamma_{i+1} = 0.65\gamma_{\text{max}}$.

6) Check for convergence of the solution

$$\left| 1 - \frac{\gamma_i}{\gamma_{i+1}} \right| < Tol \quad \text{the solution has converged.} \quad G = G_i \text{ and } \varepsilon = \varepsilon_i.$$

$$\left| 1 - \frac{\gamma_i}{\gamma_{i+1}} \right| > Tol \quad \text{the solution has not converged.} \quad \text{Go to step 2) and use } \gamma_i = \gamma_{i+1}.$$

in which $Tol$ is defined as the tolerance for the solution.

In step 5 of the above procedure, the constant 0.65 represents the ratio of the average uniform shear stress to the maximum shear stress during the earthquake (Ref. 12).

### 2.3.2 Nonlinear Analysis Methods

Two different nonlinear methods are available to predict the free-field ground motion response. One method to account for the nonlinear behavior of the soil deposit can be to model the soil stiffness with an appropriate model such as a bilinear model (Ref. 9). The second method to account for nonlinear soil behavior is to use a secant stiffness method in which the shear modulus and damping factors are updated at the end of each integration time step during the response analysis. The benefit to using a nonlinear model is that energy dissipation and stiffness variations are intrinsically included in the
analysis. Any model that represents the actual stress-strain curves for the soil could be implemented in the equation of motion.

In this study, nonlinear behavior of the soil deposit was modeled with the secant stiffness method using the modulus reduction and damping coefficient curves. An iteration scheme at the end of each time step was introduced to solve for the soil response in equation 2.20. This method results in the determination of the effective shear modulus and damping ratio at the end of each time step. The primary curve for a given soil deposit can be constructed using the modulus reduction curve and plotting values of shear force and displacement defined as

\[ G = \frac{\tau}{\varepsilon} \quad (2.23) \]

\[ \varepsilon = \frac{\Delta}{H} \quad (2.24) \]

in which \( \tau \) is the shear stress in the deposit, \( \varepsilon \) is the shear strain in the deposit, and \( \Delta \) is the displacement at the top of the soil deposit. Substituting equation 2.24 into equation 2.23 and solving for the shear stress results in the force per unit area of soil column

\[ F = \frac{G_{\text{eff}}}{H} \Delta \quad (2.25) \]

in which \( G_{\text{eff}}/H \) represents the effective stiffness and

\[ G_{\text{eff}} = G_{\text{max}} (G_{\text{RED}}) \quad (2.26) \]

in which \( G_{\text{max}} \) is the low strain shear modulus and \( G_{\text{RED}} \) is a modulus reduction factor.

To solve equation 2.20 using the nonlinear method, the following procedure was used:

1) Solve equation 2.20 for the incremental displacement response, \( \Delta x_i \), using an initial value for \( G \) and \( \varepsilon \),
2) Determine an improved estimate of the displacement using the Newton-Raphson Second Method,

\[ dx_{i+1} = dx_i + \left[ \frac{f''(x_i)}{2f'(x_i)} \right]^{-1} \left( f'(x_i) - \frac{f(x_i)}{f'(x_i)} \right) \]  

(2.27)

in which \( dx_{i+1} \) is the incremental displacement for iteration \( i+1 \).

3) Define the functionals in equation 2.27 as

\[ f(dx_i) = \omega^{2*} dx_i - a^* \]  

(2.28)

\[ \omega^{2*} = \frac{6}{h^2} + \frac{6\epsilon_i \omega}{h} + \omega_i^2 \]  

(2.29)

\[ a^* = -\Delta \ddot{x} + \frac{6\ddot{x}}{h} (1 + \epsilon_i \omega h) + \dddot{x} (3 + \epsilon_i \omega_i h) \]  

(2.30)

in which equation 2.28 was developed using equation 2.20 and assuming a linear acceleration distribution.

\[ f'(dx_i) = \frac{1}{60h} [-f(dx_i - 3h) + 9f(x - 2h) - 45f(x - h) + 45f(x + h) - 9f(x + 2h) + f(x + 3h)] \]  

(2.31)

\[ f''(dx_i) = \frac{1}{180h^2} [2f(x - 3h) - 27f(x - 2h) + 270f(x - h) - 490f(x) + 270f(x + h) - 27f(x + 2h) + 2f(x + 3h)] \]  

(2.32)

in which the functional expressions in equation 2.31 and equation 2.32 are derived using a central difference evaluation for the first and second derivative, respectively (Ref. 10).

4) Check convergence of the solution.

\[ |dx_{i+1} - dx_i| < Tol \] then solution has converged, continue using step 1)

\[ |dx_{i+1} - dx_i| > Tol \] then use \( dx_{i+1} \) in step 2) and repeat.
2.3.4 Model Verification

To verify the ground response model, two different approaches were taken:

1) Develop the primary curve using the equations 23 through 26 and compare with the force-displacement response from the computer model using the nonlinear method.

2) Compare the ground displacement response history using the equivalent linear method and nonlinear method.

The soil deposit was assumed to be 100 feet deep and use the modulus reduction and damping factors shown in Figures 2.6 and 2.7. For all cases, the Northridge earthquake was used as the bedrock ground motion, and the soil deposit was assumed to be sand using $G = 2,500$ ksf. Excellent correlation of the theoretical primary curve in Figure 2.8 and the force-displacement response of the soil deposit shown in Figure 2.9 can be noted. Agreement of the primary curve calculated using equations 2.23 through 2.26 as compared to the force-displacement response of the computer model indicate that the iteration scheme employed for the Nonlinear Method converges correctly. Figure 2.10 shows the free-field displacement time history response using the equivalent linear and nonlinear methods. Although the frequency content differs, both methods calculate the maximum ground displacement of approximately 0.12 feet. This result is expected because the equivalent linear method was developed to capture the maximum ground response. Applying a scale factor of 4.0 to the Northridge Sylmar earthquake, Figure 2.11 indicates that the equivalent linear method predicts a maximum ground displacement of approximately 50% greater than the nonlinear method. Although this is an unexpected
result, the nonlinear method should estimate lower ground displacements due to significant nonlinear behavior associated with the large bedrock accelerations.

2.4 Response Spectrum Method

Unlike the response history method in which the governing equations are solved to determine the DOF response, the Response Spectrum Method (RSM) combines the maximum response of individual modes to estimate the response because the maximum values do not occur simultaneously. To determine the dynamic displacement, $x_i$, the following mode combination methods can be used

$$x_i = \sqrt{\sum_{n=1}^{\text{Tot}} \left( \phi_{ni} \gamma_p z_n \right)^2}$$  \hspace{1cm} (2.33)

$$x_i = \sqrt{\left( \sum_{n=1}^{\text{Tot}} \sum_{m=1}^{\text{Tot}} \alpha_{nm} \left( \phi_{ni} \gamma_p \right) \left( \phi_{mi} \gamma_p \right) \rho_{nm} z_m z_n \right)}$$  \hspace{1cm} (2.34)

in which $\phi$ is the mode shape for mode $n$ at DOF $i$, $\gamma_p$ is the mode participation factor for mode $n$, $z$ is the maximum modal response for mode $n$, and $\alpha$ and $\rho$ are factors to correlate the response of mode $m$ and mode $n$. Equation 2.33 is defined as the Square Root of the Sum of the Squares (SRSS) method and equation 2.34 is defined as the Complete Quadratic Combination (CQC) method. A complete derivation of the SRSS and CQC methods are available in Ref. 3.

For non-synchronous ground motion response, the displacement at each DOF can be estimated using

$$x_{ic} = \gamma_{MF} x_{uni}$$  \hspace{1cm} (2.35)
in which $x_{ic}$ is the maximum displacement response due to incoherent ground motion, $\gamma_{MF}$ are proposed magnification factors to account for the incoherent ground motion, and $x_{uni}$ is the maximum displacement response calculated using equation 2.33 or 2.34. $\gamma_{MF}$ are empirically derived factors that are a function of the soil deposit, ground motion and structural frequency and will be discussed in Chapter 3.
CHAPTER 3
PARAMETRIC STUDIES

3.1 Introduction

Prediction of earthquake force levels in bridges is critical to properly size members to resist seismic forces. The effects of non-synchronous ground motions have not been well understood due to the nature of the response. When a bridge is assumed to be excited with uniform base motion, the total response of the structure can be determined using well-established dynamic theories. During non-synchronous ground motion, the bridge response consists of dynamic and pseudo-static support components. Bridge response due to incoherent ground motions can be predicted using a response history analysis as described in Chapter 2. However, in the state-of-the-practice, the Response Spectrum Method (RSM) is the analysis procedure that is most commonly used to evaluate bridge response due to seismic loading. Difficulty arises in the RSM to correlate not only the dynamic response of the structure due to different support ground motions, but also the pseudo-static displacement response. To explain the behavior of the non-synchronous ground motion, past researchers have used coherency function models to correlate the out-of-phase waves from different input earthquake motions. Although these models have been shown to accurately predict the correlation between different ground motions, definition of the coherency function models require an analysis using response history functions of the earthquake ground motions. If actual or synthetic time histories are available, response history analysis methods will provide the best results.

The purpose of the parametric studies is to identify different aspects of incoherent ground motions that affect bridge response. A simplified bridge model with two degrees-
of freedom (DOF) was studied to better gain an understanding of non-synchronous ground motion. This chapter will discuss a brief overview of coherency function models, frequency parameters used in the study, the derivation of the coupling ratio as a measure of structure stiffness, parameters used to model the soil columns, results from the parametric studies, and limitations of the parametric study.

3.2 Coherency Function Models

To understand and accurately solve a structure subjected to asynchronous ground motion using a RSM, coherency of the input ground motions must be defined. A coherency function model that has been proposed (Ref. 7) can be expressed as

$$\gamma_{kl}(\omega) = \frac{G_{\tilde{u}_k \tilde{u}_l}(\omega)}{\sqrt{G_{\tilde{u}_k \tilde{u}_k}(\omega)G_{\tilde{u}_l \tilde{u}_l}(\omega)}}$$

(3.1)

in which $\gamma_{kl}(\omega)$ is a complex valued function and defines the statistical dependency of the ground motions $k$ and $l$, and $G$ is the auto-spectral densities at $k$ and $l$. Interpretation of the coherency function is that when $\gamma_{kl} = 1$ the ground motions are identical and in-phase at frequency $\omega$. Equation 3.1 can be re-written as

$$\gamma_{kl}(\omega) = |\gamma_{kl}(\omega)| \exp[i \theta_{kl}(\omega)]$$

(3.2)

in which $|\gamma_{kl}|$ is the bounded modulus and varies $0 \leq |\gamma_{kl}| \leq 1$, and $\theta_{kl}(\omega)$ is termed as the phase angle between the motions at $k$ and $l$ and is defined as

$$\theta_{kl}(\omega) = \frac{\text{Im} \gamma_{kl}(\omega)}{\text{Re} \gamma_{kl}(\omega)}$$

(3.3)

in which $\text{Im}$ is the imaginary and $\text{Re}$ is the real parts of $\gamma_{kl}(\omega)$, respectively. It has been shown that $\theta_{kl}(\omega)$ characterizes the site-response effect (Ref. 7). Significant
computational effort is required to develop the coherency function using equations 3.1 through 3.3 from a site response analysis.

### 3.3 Frequency Parameters

To determine the non-synchronous ground motion effects on bridge response, a wide range of dynamic parameters were used. For a certain range of mass and stiffness properties with fundamental periods near the predominant period of the earthquake ground motion, it would be expected that the dynamic portion of the response would be dominant. In flexible structures having periods that are much larger than the predominant earthquake motions, it is expected that the pseudo-static displacements would dominate the bridge response. To quantify the range for typical bridge types, bridges with fundamental periods, \( T_p \), ranging from \( 0.5s \leq T_p \leq 3.0s \) were considered. To simplify the parameters, the period ranges studied were based on the uncoupled periods of each DOF. In each period range considered, the ratio of the periods \( T_R \) for the two DOF were varied from

\[
T_R = T_2 / T_1 = 0.1, 0.4, 0.7, 1.0
\]  

(3.4)

in which \( T_1 \) and \( T_2 \) are the uncoupled periods for DOF 1 and DOF 2, respectively. In all cases, the damping ratio, \( \varepsilon \), was assumed to be 5% and the spring support stiffness was 100 k/ft. Changing the masses at the DOF varied the period range and ratios. To account for the shear beam stiffness effect to the overall DOF response, a new parameter denoted as the Coupling Ratio (CR) was developed and will be discussed in the following section. For each period range and period ratio, CR was varied

\[
CR = 0.1, 0.4, 0.7, 1.0
\]  

(3.5)
A CR = 0 indicates that each DOF responds as a single-degree-of-freedom oscillator, while a CR = 1.0 indicates that the bridge responds as a rigid body. Mass and shear beam stiffness parameters used in the study are listed in Table 3.1 and Table 3.2, respectively.

### 3.3.1 Coupling Ratio

As mentioned in the previous section, a parameter that was developed to quantify the effect of the shear beam stiffness was the Coupling Ratio and is defined as

\[
CR = 1 - \left( \frac{T_2}{T_1} \right)_{c} \left( \frac{T_2}{T_1} \right)_{u}^{-1}
\]

in which CR is defined as the Coupling Ratio and varies from \(0 \leq CR \leq 1\), \(T_1\) and \(T_2\) are the periods of the 2 DOF systems, and the subscripts \(c\) and \(u\) denote the coupled and uncoupled period ratios, respectively. Due to the shear beam, the DOF 1 response must be solved considering the response of DOF 2. Equation 3.6 was developed based on the following observations of a two DOF system. For a system with an extremely flexible shear beam,

\[
\left( \frac{T_2}{T_1} \right)_{c} \approx \left( \frac{T_2}{T_1} \right)_{u}
\]

(3.7)

For a system with an extremely stiff shear beam,

\[
\left( \frac{T_2}{T_1} \right)_{c} \approx 0
\]

(3.8)

Equation 3.8 approaches 0 because the two DOF’s do not respond independently and approach rigid body response. Several examples representative of CR approaches 1 are:

1) In the longitudinal direction due to the high axial stiffness of the superstructure, and 2)
In the transverse direction with a high aspect ratio (width of bridge deck / length of bridge).

### 3.3.2 Ground Motions and Soil Parameters

Three different bedrock ground motions were used in the parametric study: the 1952 Taft, the 1940 El Centro, and the 1994 Northridge earthquakes. As mentioned earlier, differing soil conditions at the supports were considered as the sole source of incoherency. For each bedrock ground motion, ground motion response was predicted for soil depths ranging from 50-feet to 300-feet deep. The parametric studies limited the soil depth difference at the supports to 100-feet based on practical limits encountered in practice. Table 3.3 summarizes the soil depths considered. The nonlinear ground response model described in Chapter 2 was used to provide the most accurate prediction of the free-field ground motion response. Absolute acceleration predicted from the ground response model is compared to each the bedrock motion in Figure 3.1 through Figure 3.6. Amplification of the ground acceleration was noted for the Taft and Northridge earthquakes, but the El Centro earthquake was attenuated. Ground displacement relative to the bedrock was predicted for each earthquake and compared in Figure 3.7 through Figure 3.12. Calculation of the absolute ground displacement time history was not required to predict the pseudo-static support response of the bridge.

For all soil deposits considered, the low-strain shear modulus, \( G \), was equal to 2,500 ksf and the unit weight of soil was equal to 120 pcf. To apply the results of the parametric study for deposits with different soil properties, an equivalent soil column height, \( H_{EQ} \), can be determined using
\[
H_{EQ} = \frac{819H}{V_s}
\]  

(3.9)

in which \(H\) is the actual soil depth in feet, and \(V_s\) is the shear wave velocity of the actual deposit in \(\text{ft/sec}\). Equation 3.9 was the result of equating the natural frequency of the deposit using the actual and the parametric study parameters (equation 2.22).

### 3.4 Results

Parametric studies were conducted using the frequency parameters discussed in this chapter. Using the computer program IGMR, discussed in Chapter 2, the response histories were calculated with the different earthquake ground motions and soil depth variations. To quantify the effects of non-synchronous ground motion, magnification factors were determined using

\[
\gamma_{MF} = \frac{\delta_{IC}}{\delta_{UNI}}
\]  

(3.10)

in which \(\gamma_{MF}\) is the magnification factor, \(\delta_{IC}\) is the maximum relative displacement at DOF \(i\) due to the incoherent ground motion, and \(\delta_{UNI}\) is the maximum relative displacement at DOF \(i\) due to the uniform base motion. \(\gamma_{MF}\) is an empirically derived response coherency function that includes site amplification effects and pseudo-static support displacements. Values for \(\gamma_{MF}\) are presented in Appendix A. Non-synchronous relative displacement response can be determined with equation 2.35 for a given uniform base response. To estimate the non-synchronous response with a different earthquake ground motion, an averaged \(\gamma_{MF}\) should be used. Although the response history analyses were conducted using three specific earthquakes, certain trends can be established upon observation of \(\gamma_{MF}\):
1) Based on the average magnification factors of earthquakes, increased force levels can be expected (Figure 3.13 through Figure 3.15) due to the incoherent ground motion. For all earthquakes with characteristic periods less than 1.5 seconds, the magnification factors change rapidly and appear to be associated with the large contribution of the dynamic response component. As the dynamic response of the structure becomes smaller for larger values of $T_1$, the magnification factors tend to become constant. For the Northridge earthquake (Figure 3.15), significant frequency contents are present at higher periods that increase the dynamic displacement contribution as compared to the other earthquakes. To explain the significance of the dynamic displacement, see observation 2.

2) As the fundamental period ($T_1$) of the structure approaches the characteristic period ($T_{pe}$) of the ground motion, a uniform seismic base input will yield the greatest bridge response (Figure 3.16). This figure was developed based on the average magnification factors of all earthquakes and frequency parameters. This result can be expected based on the Dynamic Response Factor, $R$, from elementary structural dynamic theory (Ref. 3). For frequency ratios ($\omega_{pe} / \omega$) significantly less than 1, $R$ approaches 1. $R$ approaches infinity ($\varepsilon = 0$) for $\omega = \omega_{pe}$, while $R$ becomes 0 for large values of $\omega_{pe} / \omega$. In the previous discussion, $\omega$ is the natural circular frequency of the structure and $\omega_{pe}$ is the predominant frequency of the forcing function. Stiff structures would typically have increased force levels for large values of $\omega_{pe} / \omega$.

3.5 Limitations of the Parametric Study

As discussed in this chapter, an in-depth parametric study was conducted using the two DOF model. The limitations to this study are:
1) The two DOF model precludes the contribution of higher modes. In some instances, including higher modes may not affect the overall response greatly. Higher mode response will affect the dynamic contribution only, while the pseudo-static displacement response will be independent of the number of modes included in the analysis.

2) Mass variations were used to achieve the different frequency parameters while the support spring stiffness remained constant. Differing support springs would cause the influence function matrix to be unsymmetrical and the pseudo-static displacement response would be different for a given $T_1$, $T_R$ and CR.
CHAPTER 4

SUMMARY AND CONCLUSIONS

4.1 Summary

Bridge response to uniform ground motion is generally well understood and can be estimated using a response history or response spectrum method. For bridges with different input ground motions at the supports, the expected behavior is not as apparent. Response history methods are currently available to account for the incoherency, but can be difficult and cost prohibitive to use for routine bridge design. During non-synchronous ground motions, structure response consists of both a dynamic response portion and pseudo-static support displacements. Four different sources of incoherency typically considered are the incoherence effect, the wave passage effect, the attenuation effect and the site-response effect. This study was limited to incoherency due to the site-response effect. To understand the effects of incoherent ground motion on bridge response, a simplified two DOF response history computer model was developed. Ground response was estimated from a nonlinear, SDOF response model. Parametric studies were conducted using three different earthquake ground motions and soil deposit depths. Objectives of the study were 1) to provide insight into bridge response due to the incoherency from the site-response effect, and 2) to provide the ground-work for a practical response spectrum method that can account for non-synchronous ground motions.

Chapter 2 described the development and verification of the computer model used to study the site-response effects. Total response of each DOF was determined from superposition of the pseudo-static and dynamic response. Pseudo-static support
displacements can be calculated using the influence function matrix. Influence functions define the displacement at each DOF due to a unit support displacement and can be developed using stiffness matrix partitioning. Development of the equations of motion shows that only the mode participation factors need to be modified using the influence functions. A nonlinear, SDOF ground response model using shear modulus and damping factors was developed. An equivalent linear method was used to verify the nonlinear model.

Chapter 3 described the parametric studies conducted using the two DOF model. To provide insight into the complexities of non-synchronous ground motions, a coherency function model was discussed that correlates the waveforms of different input ground motions in the frequency domain. This model can be used to correlate the input ground motions of two separate functions at different frequencies. Response history analysis does not require the use of coherency functions. Parametric studies were conducted that varied the masses at the DOF, shear beam stiffness, and soil deposit depth. Three different earthquakes were selected and free-field ground motions were determined using the nonlinear ground response model described in Chapter 2. Magnification factors were calculated that compared the maximum relative displacement responses at each DOF due to the incoherent and uniform base motions.

4.2 Conclusions
Practicing bridge engineers typically use the response spectrum method to estimate earthquake-induced forces. Response history analysis is usually impractical and cost prohibitive for most bridge designs. Although this study did not result in a response
spectrum method to account for incoherent ground motions, insight into the site-response effect on bridge response was observed from the parametric studies:

1) Increased force levels were predicted in period ranges less than 1.5 seconds based on average magnification factors for three different earthquakes. Dynamic effects appeared to contribute to rapid changes in the magnification factors. When the dynamic response was not significant, the magnification factors approached a constant level. This result indicates that bridge columns would have increased ductility requirements and additional reinforcement may be required to increase the ductility capacity. Due to a potential increase in column over-strength forces, increased foundation costs could result.

2) As the fundamental period of the bridge approached the characteristic period of the ground motion, uniform base motion developed greater force levels. This result can be expected because the Dynamic Response Modification factor increases dramatically when the frequency of the bridge and ground motion are close. As a result, uniform base motion would predict maximum column ductility demands and incoherent ground motion response could be ignored.

3) As the fundamental period of the bridge exceeded the characteristic period of the ground motion, combination of different soil depths at the supports did result in amplification and reduction of forces. No apparent trends were noted.

4.3 Future Research

Additional studies that should be addressed to implement this study in bridge design practice are:
1) A response history analysis of an actual bridge should be compared with a response history analysis of an equivalent two DOF system to show the applicability of the method.

2) Additional studies need to be conducted to quantify the effect of different spring support stiffnesses. Magnification factors developed in Chapter 3 were based on varying the DOF mass instead of the support spring stiffness. In an actual bridge, an equivalent two DOF system would most likely have different masses and support springs. Different support spring stiffnesses will affect the influence function matrix and distribution of the ground acceleration and pseudo-static support displacements.

3) Higher modes of an actual bridge that could be excited during the incoherent earthquake motions should be studied to determine the possible limitations of the two DOF model.

4) Based on the parametric study results, determine the contribution of the dynamic and pseudo-static responses to the overall response. Separation of the effects will confirm the observations made in Chapter 3, and may simplify implementation into a response spectrum method.

5) Use the parametric study results to develop response spectrum modification factors. Site response factors for each soil deposit and earthquake can be determined and the contribution to the overall response can be calculated.

6) Use the methodology developed in item 5 to apply the site response factors to a design response spectrum. To verify the approach, develop ground motions that simulate the design response spectrum and compare response history results to the simplified method.
REFERENCES

1) AASHTO. “Standard Specifications for Highway Bridges,” 16th Ed., American Association of State Highway And Transportation Officials, Washington D.C.,


Table 2.1: Parameters used to verify the computer model

<table>
<thead>
<tr>
<th>$M_1$ (k-s²/ft)</th>
<th>$K_1$ (k/ft)</th>
<th>$K_{SB}$ (k/ft)</th>
<th>$M_2$ (k-s²/ft)</th>
<th>$K_2$ (k/ft)</th>
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<td>305</td>
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Table 3.1: Masses used for the 2 DOF model

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<th>$T_R$</th>
<th>$M_1$ (k-s²/ft)</th>
<th>$M_2$ (k-s²/ft)</th>
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### Table 3.2: Shear beam stiffnesses used in the 2 DOF model

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</thead>
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### Table 3.3: Soil deposit depth combinations

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Figure 2.1: 2 DOF Model

Figure 2.2: Pseudo-static Displacement Response
Figure 2.3: Dynamic displacement response comparison at DOF 1

Figure 2.4: Total displacement response comparison at DOF 1
Figure 2.5: Ground Motion Response Model
Figure 2.6: Variation of damping ratios for sands (from Ref. 8)
Figure 2.7: Variation of shear modulus for sands (from Ref. 8)
Figure 2.8: Theoretical primary curve for a sand deposit

Figure 2.9: Primary curve used in the computer model for a sand deposit
Figure 2.10: Ground motion response - Northridge earthquake (Scale = 1)

Figure 2.11: Ground motion response - Northridge earthquake (Scale = 4)
Figure 3.1: Predicted Absolute Ground Acceleration: Taft Earthquake

Figure 3.2: Predicted Absolute Ground Acceleration: Taft Earthquake
Figure 3.3: Predicted Absolute Ground Acceleration: El Centro Earthquake

Figure 3.4: Predicted Absolute Ground Acceleration: El Centro Earthquake
Figure 3.5: Predicted Absolute Ground Acceleration: Northridge Earthquake

Figure 3.6: Predicted Absolute Ground Acceleration: Northridge Earthquake
Figure 3.7: Predicted Relative Ground Displacement: Taft Earthquake

Figure 3.8: Predicted Relative Ground Displacement: Taft Earthquake
Figure 3.9: Predicted Relative Ground Displacement: El Centro Earthquake

Figure 3.10: Predicted Relative Ground Displacement: El Centro Earthquake
Figure 3.11 : Predicted Relative Ground Displacement : Northridge Earthquake

Figure 3.12 : Predicted Relative Ground Displacement : Northridge Earthquake
Figure 3.13: Average Magnification Factors: Taft Earthquake

Figure 3.14: Average Magnification Factors: El Centro Earthquake
Figure 3.15: Average Magnification Factors: Northridge Earthquake

Figure 3.16: Average Magnification Factors: All Earthquakes